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# ANALYSIS OF ERRORS IN SOLVING MATHEMATICAL PROBLEMS INVOLVING FRACTIONS

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***Abstract:*** **The study entitled "Analysis of Errors in Solving Mathematical Problems Involving Fractions among Grade 7 Students of Zamboanga National High School West used a descriptive qualitative research method. It identified the causes of errors prevalent among Grade 7 students of Zamboanga City High School-West in solving mathematical problems involving fractions. It also identified students' error types and patterns, as well as corrective instructions/strategies to address the errors. Of the total population of 1,997, only 333 Grade 7 students of Zamboanga National High School-West were chosen through random sampling as participants for the diagnostic examination. Stage I of the study was a diagnostic test that determined the error types and patterns committed by the participants in solving mathematical problems involving fractions. In stage 2, 8 of the 333 participants in the diagnostic examination, were purposively chosen for the interview intended to probe into the causes of errors. Newman's Error Analysis (NEA) Techniques was used to determine the participants' prevalent error types/patterns. The findings revealed that the participants' most prevalent error was transformation error which totalled 1,417 or 50.6 percent of the total errors. This was followed by procedural or basic facts error which totalled 941 or 33.6 percent. It was also found out that the underlying causes of errors committed are the following: conceptions and preconceptions to the present Mathematics classroom that was not corrected previously, teacher's pedagogical skills or teacher competence, lapses in concentration or hasty reasoning, lack of long term learning, poor exposure on such kind of word problems. Based on the findings, this study proposed corrective instructions/strategies which may be adopted by mathematics teachers in addressing learning problems related to the prevalent errors identified.**

***Keywords:*** **corrective measures, causes of error, error patterns, error types, fractions.**

## 1. INTRODUCTION

**Rationale:**

One of the pressing problems confronting our educational system today is low achievement in academics among graduates (Lapus, 2006). It was observed that the achievement level in mathematics and science among elementary and secondary school graduates was below standard. A recently published paper by Paglinawan (2014) corroborates this observation and shows a decline in the quality of Philippine Education at the elementary and secondary levels. Results of the National Assessment Test (NAT) among elementary and high school students, as well as the National Career Assessment Examination (NCAE) have been lower than expected. In these examinations, mathematics is usually the area in which students get the lowest scores. My years as an educator reveal that of all mathematical concepts, fractions remain to be one of the most difficult to learn and teach.

Students' difficulty in learning fractions was documented in the studies of Ashlock (2006), Voza (2011), Chinnappan, M. & Lawson, M. J. (2002), and Gabriel (2013). A study by Perchiasamy et al., (2014) revealed that a huge percentage of individuals lack basic fraction skills, particularly those that involve basic mathematical operations like addition, subtraction, division and multiplication. On the other hand, Yea-Ling's (2005) study found that solving problems involving fractions was more difficult than problems on decimals or whole numbers, especially for low ability students.

But it seems that such is not only true to low ability students but almost all students, especially in the elementary level. As the National Assessment of Educational Progress reports, fractions are "exceedingly difficult for children to master" (NAEP, 2001, p. 5). This is perhaps why students are unable to remember prior experiences with fractions from lower grade levels, Groff (1996) claims.

According to Van de Walle, "Fractions have always represented a considerable challenge for students, even into the middle grades" (2004: 293), which is why it is important to study the difficulties of students in dealing with fractions. Studies show that for students, understanding concepts on fraction are much more difficult than grasping whole number concepts. For instance, it is relatively easy to compare and order whole numbers which can be done through simple inspection. However, with fractions, the notion of "next" number does not apply since there are innumerable other fractions between any two fractions (Spangler 2011).

Furthermore, students struggle with questions like: "When you add fractions, why do the denominators have to be the same? And why do you add the numerators, but not the denominators? When you multiply fractions, why do you multiply both the numerators and the denominators? And why is the product of two proper fractions smaller than either of the two fractions? When you divide fractions, why do you invert the divisor and multiply? And why is the quotient sometimes greater than the dividend when dividing with fractions?" (Spangler 2011).

The capacity to understand and compute with fractions is an extremely important skill needed in most walks of life. It is also a foundational skill most needed for algebra (National Mathematics Advisory Panel 2008), a significant application of the other areas of mathematics, and a contribution to one's being a numerate person (Booth and Newton, 2012; Brown and Quinn, 2010; Cinnappan, 2002; and Wu, 2005). Moreover, Gabriel et al. (2013) found out from teachers that learning and understanding fractions seem crucial as they can lead to mathematics anxiety and affect opportunities for further engagement in math and science.

According to the National Conference in Teaching Mathematics (NCTM), if students struggle with learning fractions, teachers too feel frustrated as they seek ways to teach fractions effectively. The same frustration is shared by mathematics instructors of the Department of Maritime Education (DME) of ZCSPC, who have to deal with students with weak foundations in math, especially in fractions. This is despite mathematical ability being an essential tool for maritime work. Maritime students, therefore, need to possess a significant degree of mathematical competency and a strong foundation in mathematics that should have already been acquired in the basic education level and not upon entrance in higher education.

Malone et al. (2003) suggested that ineffective teaching could cause mathematic difficulties among students. This claim is corroborated by Isik, et al. (2012) who observed that pre-school teachers fail to ingrain concepts of fractions when they taught division of fractions. As Ambrose (2004) concluded, pre-service teachers may have to improve their conceptual understanding of fractions. Mathematical errors are common. Students of any age irrespective of their performance in mathematics have experienced getting mathematics wrong. It is natural that analyzing students' mathematical errors is a fundamental aspect of teaching for mathematics teachers (Hall, 2007).

Along this line, it is believed that a profound awareness and understanding of the errors committed by students in dealing with fractions and knowing the reasons why they commit such can help instructors craft lessons, strategies, syllabi and activities that address students' needs (Jackson, 2009). Koshy (2000) suggests, mathematical errors can provide useful insights for teachers into a child's thinking and understanding which is an effective mechanism for assessing student learning, and with sensitive handling, can enhance such learning. Reinforcing this idea is Voza (2011) who had researched on numerical reasoning and thinking and had found the importance of identifying the types of errors that students make when working on math problems involving fractions and for teachers to use the appropriate activities to re-introduce the concept.

It was in this context that this study on the "Analysis of Errors in Solving Mathematical Problems Involving Fractions among Grade 7 Students of the Zamboanga National High School- West" was created. The primary aim of this study was to identify and analyze the error patterns of ZNHS-West Grade 7 students in solving mathematical problems involving fractions, determine the causes of these errors and suggest the development of corrective instructions and strategies that would address learner needs more effectively. These are also expected to enhance students' learning and develop their mathematical competencies.

The Commission on Higher Education Memorandum Order (CMO) No. 59, series of 1996; CMO No. 59, series of 1999; CMO No. 13, series of 2013; CMO No. 32, series of 2013; CMO No. 20, series of 2015 and CMO 67, series of 2017 mandate maritime schools, including ZCSPC, to enhance maritime education so it can be at par with ISO standards. This entails the implementation of outcomes-based education which, in turn, demands intensified problem solving-based mathematics activities that include fractions. It has been observed that maritime students in ZCSPC, generally experience difficulty in solving mathematical problems involving fractions which are common in maritime courses like college algebra, trigonometry, calculus and other allied and professional courses. For this reason, the conduct of this study becomes even more compelling.

Grade 7 students were involved in the study as respondents because they are prospective college students who may enroll in the maritime programs of the ZCSPC.

**Review of Related Literature**

This section of the study presents previous studies that have identified a range of error types committed by students in solving mathematical problems involving fractions, their possible causes, and existing corrective instructions that can be developed and adapted to address the errors that this study identifies.

***Error Types***

Hodes et al. (2014) proposed four types of errors committed by students in solving mathematical problems. These are careless errors, conceptual errors, application errors and procedural errors. They are defined as follows: "careless errors are mistakes made which can be caught automatically upon reviewing one's work; conceptual errors are mistakes made when there is insufficient understanding of the properties or principles covered in the textbook or lecture; application errors are made when knowledge of concepts is present, but application to specific situations or questions is difficult; and procedural errors occur when directions are skipped or misunderstood, but an answer is arrived at anyway.

On the other hand, Newman as adopted by White (2005) proposed the following error types and descriptions: reading error occurs when a learner could not read a keyword or symbol in a word problem that it prevents him/her from proceeding further along an appropriate solving path; comprehension error occurs when a learner can read all words in a word problem but could not grasp the overall meaning of the words, thus, unable to proceed along an appropriate problem-solving path; transformation error occurs when a learner understands what the problem wants him/her to discover but is unable to identify the appropriate operation or sequence of operations needed to solve the problem; procedural or basic fact error occurs when a learner can identify the appropriate operation or sequence of operations but does not know the necessary procedures to arrive at an accurate answer; and encoding error occurs when a learner can work out the solution to the problem but could not express this solution in an acceptable written form.

A study conducted by Yusuf (2011), which analyzed the mathematical errors committed in primary schools, revealed largely procedural errors such as: **basic facts error** committed by 33.1 percent of the respondents and **grouping error** – 21.9 percent. Moderately committed errors were **defective, incorrect operation** -19.9 percent and **algorithm** – 17.9 percent. The error which was least likely committed was identity error – 1.2 percent while no error was committed by 6 percent of the respondents.

Meanwhile, Dindyal (2009) inferred that students committed **errors in comprehension and over-reliance on instrumental understanding** and that the common identifiable types of **computational error** were on all the four fundamental operations.

***Causes of Errors***

Miranda (2004) inferred that high school students committed errors in comprehension due to the poor and slow understanding of the keywords and terms used in the problem, thereby resulting in the students' inability to analyze the problem.

Likewise, terms used to describe fractions can be daunting for some students. For instance, a number can be called multiple names: "one half is also five tenths, zero point five (0.5) and fifty percent (50%); as well as two quarters, three sixths, four eighths and so on". Moreover, various words refer to fractions when used in different ways. "There are improper fractions and mixed numbers as well as common fractions that may have common, low or lowest denominators and be equivalent or irrational" Kaur (2004) and (Hurrell 2013). This could contribute to the difficulties and confusion of both teachers and students.

Baroody (2015) suggest that students' errors in fractions may be caused not only by a poor understanding of words used in the problem but also by a poor understanding of underlying concepts. Hansen (2011) has suggested that students' desire to memorize formulas or algorithms instead of understanding the concepts was a major reason why they continued to experience difficulties with fractions. Van de Walle and Naiser (2004) assert that "the topic of fractions may be one of the most common areas in mathematics in which students are often presented with rules without justification." That is why many students can compute fractions without necessarily understanding the concept behind the operation. Moss and Case as cited by Meagher (2002), pointed out that "Although most students eventually learn the specific algorithms they are taught, their general conceptual knowledge often remains remarkably deficient."

On the other hand, Tay Lay Heong (2005) and Adeleke (2007) asserted that errors committed by students sometimes lie in their difficulties to correctly do problem analysis and appropriately apply mathematical problems in real life situations.

Lack of mathematics skills was reported by some researchers (Swan, 2008; Puchner, 2008; Berch and Mazzocco, 2007; Garderen, 2006; Osmon et al., 2006; Morales, 2014; and Nathan et al. 2002). Berch and Mazzocco (2007) who observed that a large number of students have not acquired basic mathematical skills. According to Kilpatrick, Swafford, and Bradford, "When students practice procedures they do not understand, there is a danger they will practice incorrect procedures, thereby making it more difficult to learn correct ones. When students learn a procedure without understanding, they need extensive practice so they do not forget the steps" (2001: 122–123). This is related to the finding of a study conducted by Tambychik et al. (2010) that revealed the influence of cognitive abilities such as the ability to recall, memorize and perceive in problem-solving.

Moreover, difficulties in understanding and representing fraction relationships were also found to cause errors in problem-solving involving fractions as shown in a study by Kilpatrick, Swafford and Findell, (2001). This was revealed through the drawn representations of fractions, some potentially distracting, and which naturally did not help build the deep understanding of fraction relationships.

The work of Dossey, Mullis, Lindquist, and Chambers (1988) as cited in Spangler (2011) shows the significance of taking student dispositions into account, based on various national assessments. Their study revealed that students who like mathematics and realize its usefulness and relevance attained higher proficiency scores than students with more negative perspectives. The study also revealed that interest in mathematics appears to decline as students went up to higher grade levels.

On the other hand, Woong KhoonYoong (2000) found other causes for the commission of mathematical errors among secondary students in Brunei and these are: different meanings students attached to concepts, incomplete or fuzzy thinking, mixed up rules, the absence of salient features, and a conformist attitude that prevents them from asking clarification questions. It can be observed that all these errors are largely related to students' disposition/attitude issues.

Researchers do not discount teacher factor in dealing with students' mathematical difficulties. Malone et al. (2003) suggested ineffective teaching as one of the reasons why students commit mistakes in solving math problems. A study by Isik et al. (2012) found out that pre-service teachers ignored the conceptual dimension of the division operation in problems regarding the division of fractions. Meanwhile, Hiebert (2009) explored the difficulty in building conceptual understanding after a procedural approach has been espoused. In a study by Ambrose (2004), it was revealed that pre-service teachers may find improving their conceptual understanding of fractions challenging, especially if their experiences of learning mathematics consisted mostly of memorizing procedures, which may have been the case.

Chinnappan et al. (2002) examined the quality of understanding developed by young children in the area of fractions and decimals. The results showed a great disparity in Year 3 children's knowledge of base fractions compared to expectations based on the K-6 curriculum. If the teaching and learning process is not equally effective for all students, the difficulties in acquiring mathematical skills could worsen. In this regard, Tambychik et al. (2010) strongly suggest that understanding students' mathematical difficulties is imperative. Wong Khoon Yong (2000) pointed out the importance of understanding how students think and the way they justify their answers.

***Imperative: Inquiry into Errors Coupled with Interview***

Borasi (1994), as cited by Wong KhonYoong (2000) suggests that students' misconceptions can be used as 'springboards for inquiry.' Bray said that teachers "would benefit from a greater awareness of common student errors and how these errors are related to key mathematics concepts" (2011: 35). Diagnostic tests can help in this awareness-raising which allows teachers to formulate strategies better (Fuchs 2000). As Siegler (2013) has found, errors, through their systematic quality, point to the particular cause of the problem that needs to be overcome.

Spangler (2011) proposed a framework for corrective instructions which have been found, through research and classroom experience, to be effective in significantly minimizing student errors in solving mathematical problems, particularly fractions. It includes accessing language, enhancing conceptual learning before procedural mastery, using manipulative representation, introducing alternative algorithms and mindfulness training, enhancing metacognition and introducing instructional games and technology.

## 2. THEORETICAL FRAMEWORK

*Figure 1* is the schematic representation of the theoretical framework of the study. It shows that the study aimed to analyze the errors committed by Grade 7 students of Zamboanga National High School – West in solving mathematical problems involving fractions, identify the errors types and patterns they exhibit, determine the causes of errors, and propose corrective instructions to address the causes. As Barker et al. (2006) found, "Part of the process of learning and solving problems include making errors that, if examined, can lead to further mathematical insight."

Error type refers to the type of mistake committed by a student in solving a particular mathematical problem. Error pattern, on the other hand, refers to the error type that is committed by a student prevalently in a number of mathematical problems. Likewise, the error pattern exhibited by Grade 7 students was determined by identifying the error type that the students, as a group, prevalently committed based on the results of the diagnostic examination.

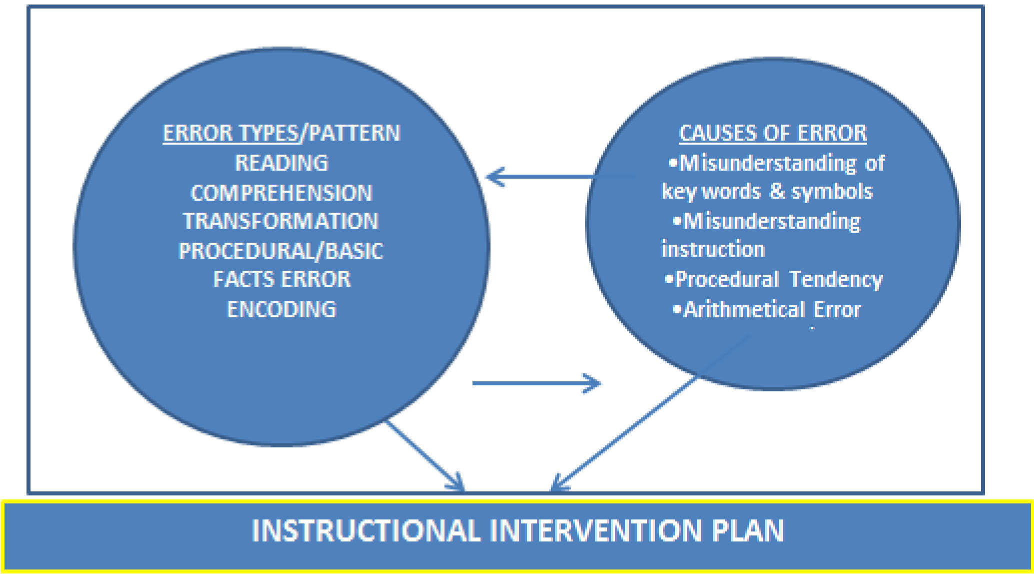


**Figure 1: The Theoretical Framework of the Study**

The theoretical framework of the study is anchored on three frameworks: **Newman's Error Analysis (NEA)** which provided the five error types used in this study; Wijaya's **Data Analysis Rubric** (2014) that provided a basis for identifying the error types committed and their causes; and Ragma's **Validated Instructional Intervention Plan** (2014) to appropriately address the causes of errors.

**Newman's Error Analysis (NEA)** classifies errors into five: reading error, comprehension error, transformation error, procedural or basic facts error and encoding error. These error types are represented by the circle in the center of the upper box in Figure 1. M. Anne Newman's (1977) theory of errors and error categories maintains that when students attempt to answer standard written mathematical questions, they have to go through a series of hurdles, namely: Reading (or Decoding), Comprehension, Transformation, Procedural or Basic facts and Encoding. According to the theory, the reading errors are committed when someone is unable to read a key word or symbol in the written problem which prevents him/her from writing his/her solution or proceeding further along an appropriate problem-solving path. Comprehension errors are committed when someone could read all the words in the question but is unable to grasp the overall meaning of the words, thus, could only indicate partially the givens of an unknown problem. Transformation is committed when someone understood the questions but is unable to identify the operation or working equation needed to solve the problem. Procedural or basic fact errors are committed when someone identifies an appropriate operation, but does not know how to proceed with it. Encoding errors are committed when someone correctly works out the solution, but could not express this solution in an acceptable written form. In some case, the answer is not in its accepted simplified form and does not indicate the unit. Researchers who made use of the abovementioned theory were Clement (2002), Ashlock (2006), Hall (2007) and Egodawatte (2011). Their studies found specific error categories. Newman's Error Analysis/Interview Protocol, as propounded by White (2005) provides a framework for identifying the causes behind students' difficulties with mathematical word problems.

Wijaya's **Data Analysis Rubric** (2014) provides a basis for the identification of the causes of errors and these are: confusion and misunderstanding of keywords and symbols for reading error; misunderstanding instruction and error in selecting information for comprehension; too much account of the context and/or wrong mathematical operation/concept for transformation error; arithmetical error, wrong algorithm and/or incorrect steps for procedural or basic facts error; and inappropriate solution, unfinished answer/unreliable answer for encoding error. These causes of errors are represented by the circles surrounding the circle at the center of the upper box, with arrows pointing towards the circle at the center, indicating that they cause the occurrence of the different error types.

The framework is represented by the box labeled **"Proposed Corrective Instructions/Strategies"** on the lower portion where the corrective measures are. The Intervention Plan proposed in this study addresses the errors of the participants to help improve their performance in mathematics.

**Statement of the Problem**

The study sought to determine the causes of the errors prevalent among Grade 7 students of Zamboanga National High School-West in solving mathematical problems involving fractions.

Specifically, it aimed to answer the following questions:

1. Which of the following error types are committed by the Grade 7 students in solving mathematical problems involving fraction:
   - 1.1 reading error
   - 1.2 comprehension error
   - 1.3 transformation error
   - 1.4 procedural or basic facts error
   - 1.5 encoding error
2. What is the error pattern exhibited by Grade 7 students of ZCNHS-W in solving mathematical problems involving fractions?
3. What are the underlying causes of mathematical errors involving fractions?
4. What corrective instructions/strategies may be developed to address the causes/s of the prevalent error types identified?

**Scope and Limitation of the Study**

The study focused on analyzing the error types and the identification of the error pattern in solving mathematical problems involving fractions of Grade 7 students of the Zamboanga National High School-West enrolled in the school year 2016-2017. It is further limited to identifying the causes of the error patterns and proposing appropriate corrective instructions/ strategies that may address the identified error types.

The error types considered were limited to those identified by Newman as adopted by White (2005) and these are reading error, comprehension error, transformation error, procedural or basic facts error, and encoding error.

The possible causes are those proposed by Wijaya (2014) which correspond to the error types classification of Newman as adopted by White (2005). These are: confusion with and misunderstanding of keywords and symbols for reading error; misunderstanding instruction and error in selecting information for comprehension error; procedural tendency, too much account of the context, wrong mathematical operation/concept for transformation error; arithmetical error, wrong algorithm, incorrect steps for procedural or basic fact error; and inappropriate solution, unfinished answer/ unreliable answer or encoding error.

A Validated Instructional Intervention Plan adapted from Ragma (2014) was proposed as possible corrective instructions/strategies to address the error pattern identified in mathematical problem-solving involving fractions and to improve their performance in mathematics.

## 3. RESEARCH METHOD

This chapter presents the research design, sources of data, data analysis, parts of the instructional intervention plan and ethical considerations. Moreover, the roles of the facilitator and researcher were included as this is an important distinguishing factor for quality in data analysis.

**Research Design**

The study employed a qualitative descriptive research design. Leary (2010) defines such design as one that includes all studies that purport to present facts concerning the nature and status of anything. This design is appropriate for the study since it seeks to provide a description of the conditions or errors of the respondents.

Furthermore, the data collection involved a diagnostic examination, interview, and observation data. Interpretation was based on a combination of researcher and facilitator's perspective and data collected with the aid of Newman's Error Analysis Technique (NEA). This approach is apt for this study since it makes use of qualitative techniques to analyze errors. To identify error patterns and the causes of errors, an in-depth interview was conducted to 8 respondents who were purposely chosen out of the 33 participants who took the diagnostic examination.

**Research Participants**

The population of this study was composed of Grade 7 students of the Zamboanga City National High School-West enrolled in the school year 2016-2017. The grade level has a total population of 1,997. Using Slovin's formula, 333 participants were chosen to take the diagnostic test

**Table 1: Demographic Profile of Participants in Terms of Sex and Age**

| **Demographics** | **Participants** | |
|---|---|---|
| **Sex** | **F** | **%** |
| Male | 130 | 39 |
| Female | 203 | 61 |
| *Total* | *333* | *100* |
| **Age** | **F** | **%** |
| 11 years old and below | 18 | 5 |
| 12-15 years old | 283 | 85 |
| 16-19 years old | 30 | 9 |
| 20 years old and above | 2 | 1 |
| *Total* | *333* | *100* |

Table 1 reveals the demographic profile of the participants in terms of sex and age. Of the 333 participants, 203 (61 percent) were females and 130 (39 percent) were males. Data also show that 283 participants (85 percent) were aged 12-15 years, 30 (9 percent) were 16-19 years, 18 (5 percent) were 11 years old and below, while only 2 (1 percent) were 20 years old and above.

The 333 participants were selected from the 40 sections of Grade 7 students through random sampling using the fishbowl method. The names of the students were written in a piece of paper, rolled and placed in a container from which 333 were chosen to participate in the study.

For purposes of coding, each of the participants was assigned a code number. The first name that was picked was given a code number of Participant 1, the next Participant 2 and so on.

As part of this study's ethical considerations, the researcher explained to the participants, in the presence of the mathematics teachers and guidance counselor, the purpose of the study and gave them the assurance that their identities shall remain confidential and that the results of the diagnostic examination will not affect their grades. The students agreed to participate in the study.

To determine the possible causes of the participants' mathematical errors, 8 respondents were purposively chosen for the interview. The interview was meant to probe deeper into the causes of the students' error types. Twenty respondents got a score of 0 in the diagnostic exam. Among them, 9 answered all the items, while the rest left some items unanswered. The ones chosen for the interview were also those with the most number of errors in the exam. Sauro (2008) said that 5-25 respondents would suffice for the conduct of an interview in a qualitative study. An informed consent was given to the 9 who were interviewed, however, only 8 came. The facilitators of the interview were mathematics professors of the ZCSPC who are currently finishing their doctoral degrees.

**Research Instruments**

Four instruments were used by this study to address the research questions and these are:

1. The **mathematics problem solving diagnostic examination** was used in identifying the error patterns commonly exhibited by individual participants and the prevalent error pattern of the group. It was constructed by the researcher and consisted of 10 mathematical word problems involving fractions.

2. The **Newman Error Analysis/Interview Protocol** adopted by White (2005) was used during the interview

3. The **Interview Summary Table** was utilized by the facilitators during the interview to document the results of the interview.

4. The **Data Analysis** was used to determine the prevalent error types and the causes of errors. It was adopted from Wijiya (2014). The rubric has three (3) components which are error types, causes of error and the explanation/s for each error.

**Data Gathering Procedure**

The researcher will ask permission to conduct the study and Conduct Interview

**Data Analysis**

Data collected were subjected to qualitative descriptive analyses and interpreted at 4 stages. In Stage 1, the researcher, with the assistance of 4 facilitators checked the answers of the participants in the diagnostic exam. Every wrong answer was analyzed. The **Data Analysis Rubric (Appendix B)** guided the analysis. The error types identified per test item for each participant were plotted and summarized in the table titled, **Error Type Exhibited by Individual Participants** (Appendix D) to show the prevalent error types of the respondents.

In Stage 2, error patterns were identified. This was done by simply adding the number of times an error type was committed as shown in the table titled, **Error Type Exhibited by Individual Participants**.

In Stage 3, the **Data Analysis Rubric** (Appendix B) guided the facilitators and the researcher in identifying and analyzing the underlying causes of the error types. The interview was conducted to reveal underlying misconceptions on the concept of fractions and confusions on its operations. The responses and the facilitator's observations and analysis were systematically recorded in the **Interview Summary Table** (Appendix F).

In Stage 4, **Corrective Instructions/Strategies,** also called "the **Instructional Intervention Plan**" were recommended to address the cause/s of the prevalent error type identified.

**Parts of the Instructional Intervention Plan**

The instructional intervention plan contains the error categories, the proposed instructional strategies, the procedures of implementing the strategy and the assessment strategy.

The instructional intervention plan is based on the participants' competencies in solving mathematical problems involving fractions.

**Statistical Treatment of the Data**

To answer the research questions, the following statistical tools were employed:

**Frequency Count and Percentage.** It was used to determine the number of students who exhibited the error types and patterns. It was also used to describe the characteristics of the participants in this study.

**Kuder Richarson Formula 20.** This was utilized to determine the reliability of the diagnostic exam involving fractions, which was administered in the pilot test/dry run among Grade 7 students of Western Mindanao State University.

## 4. RESULTS AND DISCUSSION

This chapter presents the analysis and interpretation of data on the respondents' errors types and patterns as well as the causes of these errors. Proposed corrective instructions/strategies were also presented.

**Problem 1. Which of the following error types in solving mathematical problems involving fractions do the respondents commit: reading error, comprehension error, transformation error, procedural or basic facts error and encoding error?**

Table 2 shows a total of 2,803 errors in the solution sheets of the 333 participants. All of committed at least 1 type of error. Almost all of them have overcome the reading error. They were able to read the mathematical keywords or key symbols from the given problems involving fraction, which could be because the word problems were made as simple as possible so that errors due to faulty handling of fractions would be revealed rather than due to difficulty in understanding the problems. A few committed reading errors since they left some items, many of which were items 8 to 10, unanswered. Most of them responded to the problems by writing some meaningful responses, albeit the ability to read is not always tantamount to comprehension. The respondents could not successfully transform, process and encode the solutions. Because of this, they were prone to manipulate the figures in the word problems.

Furthermore, the solutions to items 1, 2 and 3 are similar that it requires the addition of unlike fractions to be able to arrive at the correct answer. Most of them knew that they just needed to add the fractions, however, they failed. Most of them committed the procedural error. For item 1, there were 168 errors, for item 2 220, and item 3 187. It is worth noting that at their grade level, this type of error is still prevalent, which is contrary to the findings of Yusuf (2011) who said that this error type was mostly identified among primary schoolers only. Although the mathematical problems given to the participants were more complicated, it can be said that most of the Grade 7 students lack the mastery of adding unlike fractions. Nevertheless, for both studies, the participants were able to choose the appropriate mathematical operation but were unsuccessful in completing the solution.

Moreover, at least 50.6 percent of the errors identified were transformation error, which were mostly exhibited in items 4 to 10. This result agrees with the study of Wijaya et al. (2014) that identified transformation as one of the most common errors committed by students. Generally, the participants were able to read and understand the problem, but not know how to proceed with the solution. They understood what the problem wants them to find out but are unable to identify the appropriate and necessary operation or sequence of operations, hence, could not proceed with solving the problem.

Moreover, 33.6 percent of the errors were procedural and this type of error was mostly exhibited in items 1 to 3. However, in the study of Yusuf (2011), this error type was mostly identified among primary schoolers only. This is because the kind of mathematical problems given to the participants in this study was more complicated than of Yusuf's study. Nevertheless, for both studies, the participants were able to choose the appropriate mathematical operation but were unsuccessful in completing the solution correctly.

Comprehension error was found to be moderate in this study, where a little over 14.4 percent of the errors committed were comprehension in nature. The participants were able to read through word problems but could not grasp the overall meaning of the words, making them unable to proceed along an appropriate problem-solving path.

Baroody (2015) affirms that students' errors in fractions may be caused by poor understanding of underlying concepts. For example, in this study, the solution for item number 4 requires subtraction of two dissimilar fractions, $\frac{3}{4} - \frac{5}{8}$ which is equal to $\frac{1}{8}$. Of the 241 errors, 143 were the failure to correctly identify the correct operation to be used in the problem, which indicates that they committed transformation error. The same type of error was also prevalent for items 5 to 10. Many students compute fractions found in the problems without necessarily understanding the concept behind the operation. Moss and Case, cited by Meagher (2002), said, "Although most students eventually learn the specific algorithms they are taught, their general conceptual knowledge often remains remarkably deficient."

**Table 2: Error Types Exhibited by the 333 Participants**

| *Type of Error Committed* | *Item Number* 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | *Total* | *%* | Rank |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Reading | 0 | 0 | 0 | 1 | 0 | 2 | 3 | 6 | 8 | 6 | 26 | .9 | 4th |
| Comprehension | 1 | 19 | 8 | 9 | 22 | 26 | 60 | 76 | 80 | 104 | 405 | 14.4 | 3rd |
| Transformation | 31 | 58 | 61 | 143 | 186 | 154 | 187 | 206 | 201 | 190 | 1417 | 50.6 | 1st |
| Procedural | 168 | 220 | 187 | 87 | 56 | 100 | 42 | 29 | 32 | 20 | 941 | 33.6 | 2nd |
| Encoding | 2 | 3 | 2 | 1 | 3 | 1 | 1 | 0 | 0 | 1 | 14 | 0.5 | 5th |
| Total Errors Identified | 202 | 300 | 258 | 241 | 267 | 283 | 293 | 317 | 321 | 321 | 2803 | 100 | |

**Problem 2.What is the error pattern exhibited by Grade 7 students of ZNHS-W in solving mathematical problems involving fraction.**

**Problem 3. What causes the mathematical errors involving fractions of the Grade 7 students of the Zamboanga National High School-West?**

To have a deeper analysis of the causes of the error types committed by the 333 participants, 8 respondents were purposely chosen for an in-depth interview with the facilitators. The interview was meant to probe deeper into the causes of the error types committed. Sauro (2008) suggests that 5-25 respondents is enough for the conduct of an interview in a qualitative study.

Table 4 reveals the summary of the error types exhibited by the individual respondents. As revealed in the table, for item no. 1and 6, 6 out of 8 respondents committed procedural errors. Most of them were able to overcome the transformation error, by identifying the correct operation, which is addition; however, 6 of them fail to add the fractions correctly$\frac{3}{4}+\frac{1}{8}$. Some added the numerators and added the denominators, such as: $\frac{3}{4}+\frac{1}{8}=\frac{4}{12}$, while adding the numerators and copied the bigger the denominator, $\frac{3}{4}+\frac{1}{8}=\frac{4}{8}$. For items no. 2 and 3, most of them did the same thing. When they were asked why they came up with the solution, they replied:

*"Ganun ang dapat gawin, Sir. Parang ganun tinuro sa amin dati. Hindi ko talaga naintindihan noong nagtuturo si Sir kasi po parang nakukulangan sa mga binibigay na example si Sir. Mabilis din po magturo si Sir."*

(That's the way it is, Sir. This was how things were taught to us. I couldn't understand our teacher's instruction for we weren't given enough examples. He was also fast with the discussion.") --*R#1, 10/10/16*

*"Ganun ang pag-intindi ko sa turo ni Sir namin dati. Nahihirapan po ako sa pag add ng fraction na magkaiba ang denominator. Ganun ang naalala kong paraan sa pag-add ng fraction na magkaiba ang denominator."*

("This was how I understood our teacher's instruction. I had difficulty adding fractions with different denominators. This is how I remember adding fractions with different denominators.")--*R#5, 10/10/16*

"*Ganun po kasi yun naaalala kong pag-add sa fraction dati. Ganun po kasi ang naaalala ko noong nageexplain si Sir. Ganun ko po naintindihan un explanation ni Sir dati. I add ko lang po un numerator at pagkatapos inadd ko din po lahat ng denominator. Pagkatapos po nireduce ko po sa lowest term yung final answers ko po*" .

(This is how I remember addition of fractions. This is how I recall and understand our teacher's explanation. I simply add the numerator and denominator then reduce the final answer to lowest term.)---*R#3, 10/10/16*

This indicates that students bring with them misconceptions and preconceptions that were not previously corrected. Learners' constructed knowledge is dependent on the cognitive structures they previously developed (Battista, 2001). In other words, their prior knowledge in solving fractions is one of the causes why they continue to commit such errors.

For Problems 5, 7, 8 and 9, all of the respondents committed transformation error. They failed to identify the correct operation, which is subtraction, for Items 4, 5, 6 and multiplication for Item 7. For Item 4, some attempted to add the two fractions while others multiplied. Most of the respondents also added $\frac{2}{3}+\frac{5}{6}$ and $12\frac{1}{2}+2\frac{5}{8}$, for items 5 and 6, respectively. When they were asked how and why they came up with the solution, they replied:

*"Ganyan ang pag-intindi ko saturo ni Sir namin dati. Nahihirapan po*

*ako sa fraction. Mabuti-buti po ako kung whole number ang isosolve ko.*

("This is how I understood our teacher's explanation. I find solving fractions difficult and am better off with whole numbers.")---*R# 6, 10/10/16*

*Ganun po kasi pagkakaaintindi/pagkakaalam ko sa pagadd sa fraction*

*noong nagtuturo sa amin dati pa. Kulang po siguro yung example dati ng*

*titser namin kasi naman po ang dami ng gawain ng titser namin,madalas*

*wala po siya sa klase at may mga times nagaabsent din po ako sa klase.*

("This is how I understood the addition of fractions. The examples given to us may have been insufficient because our teacher was frequently out causing us to also skip class.)--*R# 4, 10/10/16*

*"Ganun ang naalala kong paraan sa pag subtract ng fraction na magkaiba*

*ang denominator. Madalas may mga assignment kami tungkol sa fraction*

*pero hind na din yun nasasagutan pagsumunod na meeting namin kay Sir"*

("This is how I remember the subtraction of fractions with dissimilar denominators to be. We often had home works on fractions, which were not discussed in subsequent meetings.")---*R#2, 10/10/16)*

*"Nakalimutan ko na paano tinuro ni Teacher. Kulang yung examples*

*binibigay sa amin about sa problem solving sa fraction. Hindi kami*

*nakakapagdiscuss ng mabuti sa klase kasi nga po madami kami sa klase,*

*maingay mga kaklase ko.*

("I've forgotten the lesson. We were given insufficient problems on solving fractions. We were unable to discuss it thoroughly because we were cramped in class and couldn't focus on the discussion.)---*R#8, 10/10/16*

The replies of the respondents indicate that teacher's pedagogical skills or teacher competence contributed to the errors committed by the respondents. Malone et al. (2003) mentioned ineffective teaching as one of the reasons why students mistakenly solve math problems.

Another cause of the errors was due to lapses in concentration or hasty reasoning.The findings of the current study adhere to the study of Dossey, Mullis, Lindquist, and Chambers (1988) as cited in Spangler (2011) which showed the need to consider students' disposition to learning.

This was evident in the responses of the interviewees:

*"Hindi ko talaga maintindihan ang fraction. Tinuro yan sa amin, Sir,*

*dati, pero nakalimutan ko na. Nakakalito ang pag solve sa fraction. Mabilis din magturo si Sir. Kulang din yung mga samples ni Sir na binibigay ".*

("I really can't understand fractions. It was taught to us, but I've forgotten about it. Solving fractions is confusing. The teacher is fast with the discussion and gives insufficient examples.")---*R# 1, 10/10/16*

*"Hind ko alam, Sir. Basta ganun sya, Sir. How many man yung tanong, so e add lang. Ganyan ang pag-intindi ko sa turo ni Sir namin dati. Nahihirapan po ako sa fraction na magkaiba ang denominator. Hindi ko alam, Sir."*

("I don't know, but that's the way it was taught to us. Simply add. I have difficulty solving fractions with different denominators.")---*R# 8, 10/10/16*

*"Ganun ang naalala kong paraan sa pag subtract ng fraction na magkaiba ang denominator. Madalas may mga assignment kami tungkol sa fraction pero hindi na din yun nasasagutan pagsumunod na meeting namin kay sir".*

("This is how I remember the subtraction of fractions with dissimilar denominators to be. We often had home works on fractions, which were not discussed in subsequent meetings.")---*R#2, 10/10/16)*

Another main cause of the error was due to lack of long-term learning. They appear to understand the concept of fractions at the end of the unit, but did not retain it. Students with long-term learning do not forget the acquired knowledge and are able to apply it in real life situations. Replies in the interviews were as follows:

*"Meron po pero hindi ko naintindihan o nakalimutan ko na. Nakakalito din po talaga ang proseso sa pagsolve sa fraction. Sa apat na operation sa pagsolve sa fraction, pareparehong mahirap intindihin"*

("I find the procedure in solving fractions confusing. All four operations are difficult to understand.")---R# 1, 10/10/16

*"Tinuro yan sa amin, Sir, dati, pero nakalimutan ko na." Nakakalito ang pagsolve sa fraction. Hindi ko maintindihan ang fraction*

(Fractions were taught to us, but I've already forgotten about them. I can't understand fractions.")--- (R# 1, 10/10/16)

Some respondents committed comprehension errors on Items 8 and 10 and simply wrote their answers without indicating what they represent. Based on this interview excerpt, the respondents had poor exposures to solving word problems. They said,

*"Hindi ko ma gets yung problem. Ganun po kasi pagkakaaintindi pagkakaalam ko sa pagadd sa fraction noong nagtuturo sa amin dati pa. Kulang po siguro yung example dati ng titser namin kasi naman po ang dami ng gawain ng titser".*

("I can't understand the problems. This was how I understood the addition of fractions to be. The examples were insufficient since our teacher was overloaded with work.)---*R# 5, 10/10/16*

*"Mahirap naman yung problem. Hindi ako sanay basta word problems.*

*Ganun ang naalala kong paraan sa pag subtract ng fraction na magkaiba ang denominator. Madalas may mga assignment kami tungkol sa fraction pero hindi na din yun nasasagutan pagsumunod na meeting namin kay Sir"..*

("Word problems are difficult. This is how I remember the subtraction of fractions with different denominators to be. We often had assignments on fractions, but they were not discussed in the subsequent meeting.")

(R#2, 10/10/16)

*"Kasi po, Ma'am, nakakalito talaga ang fraction. Mahirap po siyang icompute lalo na at word problem. Siguro maam kulang pa rin yung mga example ni titser".)*

("Fractions are confusing especially in word problems. Perhaps we were not given enough examples.")---(R# 6, 10/10/16)

**Table 3: Summary of Error Types Exhibited by Individual Respondents**

| Item | Respondent Number | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 1 | **P** | **P** | **T** | **T** | **P** | **P** | **P** | **P** |
| 2 | **P** | **P** | **P** | **T** | **T** | **P** | **P** | **P** |
| 3 | **P** | **P** | **P** | **T** | **P** | **P** | **P** | **P** |
| 4 | **P** | **P** | **T** | **T** | **P** | **P** | **P** | **P** |
| 5 | **T** | **T** | **T** | **T** | **T** | **T** | **T** | **P** |
| 6 | **P** | **P** | **T** | **T** | **P** | **P** | **T** | **P** |
| 7 | **T** | **P** | **T** | **T** | **P** | **T** | **T** | **T** |
| 8 | **T** | **T** | **T** | **T** | **T** | **T** | **T** | **T** |
| 9 | **T** | **T** | **T** | **T** | **T** | **T** | **T** | **T** |
| 10 | **R** | **R** | **C** | **R** | **C** | **R** | **R** | **C** |

*Legend:*

***R** - Reading Error* *__C__ - Comprehension Error*

***T** - Transformation Error* ***P** - Procedural Error* ***E** - Encoding Error*

**Problem 4. What possible corrective instructions may be developed to address the cause/s of the prevalent error types identified?**

The participants' poor performance in solving fractions can be attributed to different factors. Corrective instruction is deemed a way to address this. The instructional intervention plan is based on the students' performance, capabilities, and error categories. It addresses error categories that will serve as a comprehensive guide for teachers in the improvement of their performance. This instructional intervention plan was adopted from Ragma (2014), with modifications created to fit the prevalent errors of the participants.

Although the most prevalent error was transformation error, the proposed intervention plan would emphasize on all error categories. A particular error may be addressed together with others. Other types of error should not be disregarded if one wants to address a particular type of error category. Hence, this intervention plan follows a step-by-step procedure addressing all types of errors in solving word problems. It started with reading error, comprehension error, transformation error, procedural error or basic facts error and lastly the encoding error. This follows the five interview protocol in identifying the types of error in solving mathematical written word problems as classified by Newman's Error Analysis (NEA), as founded by White (2005). The plan suggests activities that address the students' error types.

**Proposed Corrective Instructional Intervention Plan in Solving**

**Mathematical Problems Involving Fractions**

The Proposed Instructional Intervention Plan was arranged in three columns. The first column was labelled as the error types committed by a student in solving written mathematical word problems and it follows the Newman's Prompt. The Newman's Prompts is a framework for teachers that identifies the specific areas where students' understanding breaks down when solving problems. This information can then be used by teachers to support student understanding. Newman's research (1977, 1983) as propounded by White (2005) said that when students attempt to answer standard, written, mathematics word problems, they go through 5 sequential hurdles: Level 1 Reading , Level 2 Comprehension, Level 3 Transformation, Level 4 Procedural or Basic Facts, and Level 5 Encoding. The second column of the plan was labelled as the Corrective Instructions, Process, Activities that teachers can use to address the types of errors committed by the students. It includes activities for Teacher-Student interactions. Every error type is provided with two activities/ strategies that a teacher may choose from. The third column contains justifications/ reasons why such activities can enhance performance. The justifications were based on the context of the participants.

**General Objectives**

The Instructional Intervention Plan is formulated to:

1. Improve the performance of students in solving mathematical problems involving fractions; and
2. Address the different errors exhibited by the students in solving mathematical problems involving fractions.

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## 5. SUMMARY, CONCLUSION AND RECOMMENDATION

This chapter presents the summary of the findings, conclusion and recommendations of the study.

**Summary of Findings**

This study aimed to determine the causes of errors prevalent among Grade 7 students of ZHNS-W in solving mathematical problems involving fractions. Specifically, it intended to identify the error types most commonly committed by the students. It also aimed to suggest corrective instructions to address appropriately the causes of errors identified.

The findings indicate that:

1. The most commonly committed mathematical errors involving fractions committed by Grade 7 students of ZNHSW were transformation error, procedural or basic facts error followed by comprehension error. Of 2,803 incorrect answers, transformation error totaled 1417 (50.6 percent) while procedural or basic facts error totaled 941 (33.6 percent). This was followed by comprehension error was committed 405 time (14.4 percent).

2. The interview revealed the following causes of errors:

This study indicates that students bring with them conceptions and preconceptions to the present Mathematics classroom that was not corrected previously. The teacher's pedagogical skills or competence also contribute to the errors committed by the students. The latter did not receive quality instruction, which could have prevented the errors and their patterns.

Moreover, errors were also due to students' lapses in concentration, hasty reasoning, and lack of long-term learning. They appear to understand the concept of fractions at the end of the unit, but could not retain it. The students also had insufficient exposure to word problems.

3. The instructional intervention plan is based on the students' level of performance, their capabilities and constraints and the different error categories in solving word problems involving fractions. All the error categories are addressed in the plan since all of them were considered constraints. The instructional plan also serves as a comprehensive guide for teachers to improve their performance and that of their students.

**Conclusions**

Based on the results, the following conclusions are drawn:

1. The most commonly committed mathematical errors involving fraction by Grade 7 students of ZNHSW were transformation error, procedural or basic facts error followed by comprehension error.

2. Students bring with them conceptions and preconceptions to the present Mathematics classroom that were not corrected previously. The teacher's pedagogical skills or competence also contributed to students' mathematical errors. Students did not receive the quality instruction which resulted to the error patterns of students. The errors are attributed to lapses in concentration, hasty reasoning, and lack of long-term learning. The students appear to understand the concept of fractions, but could not sustain it. Lastly, the interview revealed that the students had poor exposure to word problems.

3. The instructional plan is intended address error categories and can serve as a comprehensive guide for teachers to improve performance and that of their students.

**Recommendations**

On the basis of the findings and conclusion, the following recommendations are drawn:

1. To address the underlying causes of errors committed by the students, it is recommended that teachers use appropriate procedures.

2. Since most of the errors committed by the participants were transformation, procedural or basic facts, and comprehension errors, a similar study of different grade levels may be done.

3. School administrators should address the problems by conducting programs to upgrade mathematics instruction from the basic education units to the higher ones.

4. Teachers should also consider students' needs and interests in their choice of pedagogy.

5. Reading and comprehension skills should be intensified in English classes.

6. The schools may adopt this study's Proposed Instructional Intervention Plan.

7. A study may be conducted to determine the effectiveness of the instructional intervention plan.

8. The Instructional Interventions Plan may be used not only in ZCHS-West but in other schools experiencing the same error patterns in solving mathemathical problems involving fractions.

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